UNIT I

Functions

Concept of function

A function f is a rule of correspondence between two sets A and B such that each element of the set A is assigned to only one element of the set B. For example, let us consider the set A as the set of all C.A. students who take the examination of the Institute of Chartered Accountants of India and set B as the set of all natural numbers. A function is defined between the sets A and B since every CA student who takes the examination has a register number which is a natural number.

From the definition of function from A to B we note that every element of A is assigned to an element of B and no element of A is left out without an assignment in B. Also no element of A can be assigned to more than one element of B. When a function is defined from A to B, the set of all elements in A is called domain and the set of all elements in B is called the codomain. If a, an element of the set A, is assigned to the element $b \in B$ by the function f, then $b = f(a)$ is called the image of a. The set of all images in B is called the range and the range is a subset of the codomain. Any function f defined from A to B is called an into function. If the range is equal to the codomain, the function is called onto function. If no two elements of the domain A has the same image the function is called a one–one function. If f is a one–one function from A to B, then for any two elements $x_1, x_2 \in A$, $x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$.

The function in which all elements of A are mapped to a fixed element of B is called a constant function.

Examples

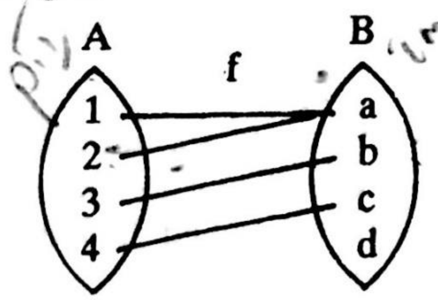


Fig 2.1

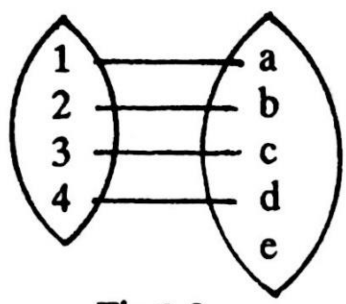


Fig 2.2

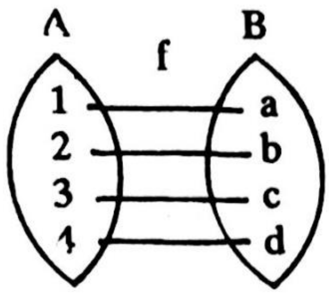


Fig 2.3

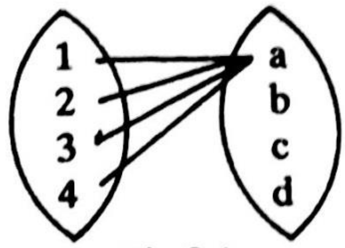


Fig 2.4

The function defined in fig (2.1) is an into function.

The function defined in fig (2.2) is an one–one function.

The function defined in fig (2.3) is an one–one onto function.

The function defined in fig (2.4) is a constant function.

The correspondence in fig (2. 5) is not a function.

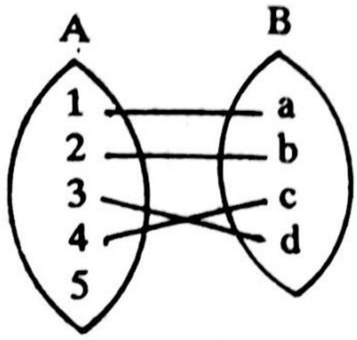


Fig 2.5

Variables and Constants

A *variable* is a quantity which can assume any value out of a given set of values. Cost, revenue, profit, height, weight etc, are examples of variables. If a variable can take values which can be put with one to one correspondence with integers, then the variable is called a discrete variable. If a variable can assume any real numbers between two real numbers a and b, then the variable is called a continuous variable. Then we say that it lies in the open interval (a,b). If x can also assume the values a and b then x lies in the closed interval [a,b]. A quantity which remains fixed is called a *constant*. Constants are represented by small letters a,b,c. . . .

If there exists a relation between two real variables x and y such that for any given real value of x there exists a value of y, then we say y is a function of x and is denoted by $y = f(x)$. x is called the independent variable and y is called the dependent variable.

Examples

$$y = x$$
$$y = 2x^2$$
$$y = 3x^2 + 5x - 8$$

Odd and Even functions

A function f(x) is called an odd function if $f(-x) = -f(x)$. A function f(x) is called an even function if $f(-x) = f(x)$

Examples

$y = x^3$, $y = \sin x$ are odd functions

$y = x^4$, $y = \cos x$ are even functions.

Example 1

Find the domain of the function

$$y = \sqrt{x^2 - 5x + 6}$$

Solution

$$y = \sqrt{x^2 - 5x + 6} = \sqrt{(x-2)(x-3)}$$

If $x < 2$ then $(x-2)(x-3)$ is positive and f(x) is real.

If $2 < x < 3$, then $(x-2)$ is positive and $(x-3)$ is negative and therefore $(x-3)(x-2) < 0$. In this case, f(x) is not defined.

If $x > 3$, then $(x-2)(x-3) > 0$ and f(x) is real.

$\therefore$ The domain of the function f is $2 \leq x \geq 3$

Example 2

Let $V = \{-2, -1, 0, 1, 2\}$. Let f be a function defined by $f(x) = x^2 + 1$. Find the range of f.

Solution

$$f(x) = x^2 + 1$$
$$f(-2) = (-2)^2 + 1 = 5$$
$$f(-1) = (-1)^2 + 1 = 2$$

$$f(0) = (0)^2 + 1 = 1$$
$$f(1) = (1)^2 + 1 = 2$$
$$f(2) = (2)^2 + 1 = 5$$

$\therefore$ The range of f is $\{1,2,5\}$

Example 3

If $f(x) = 3x^2 - 2x + 5$ find $f(-2)$, $f(2)$, $f(0)$ and $f\left(\frac{1}{2}\right)$

Solution

$$f(x) = 3x^2 - 2x + 5$$
$$f(-2) = 3(-2)^2 - 2.(-2) + 5$$
$$= 12 + 4 + 5 = 21$$
$$f(2) = 3.(2)^2 - 2.2 + 5$$
$$= 12 - 4 + 5 = 13$$
$$f(0) = 3.0 - 2.0 + 5 = 5$$
$$f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 - 2\ \frac{1}{2} + 5$$
$$= \frac{3}{4} - 1 + 5 = \frac{19}{4}$$

Example 4

If $f(x) = 2x^2 - 5x + 4$ for what values of x is $2f(x) = f(2x)$

(CA May 1986)

Solution

$$f(x) = 2x^2 - 5x + 4$$
$$2f(x) = 2[2x^2 - 5x + 4]$$
$$= 4x^2 - 10x + 8$$
$$f(2x) = 2(2x)^2 - 5(2x) + 4$$
$$= 8x^2 - 10x + 4$$

Since $2\,f(x) = f(2x)$

$$4x^2 - 10x + 8 = 8x^2 - 10x + 4$$

$$4x^2 = 4 \quad \text{or} \quad x = \pm 1$$

Example 5

If $f(x) = \sin x$ and $\phi(x) = \cos x$ show that

$$f(x+y) = f(x)\,\phi(y) + \phi(x)\,f(y)$$

Solution

$$f(x) = \sin x, \quad f(y) = \sin y$$

$$\phi(x) = \cos x, \quad \phi(y) = \cos y$$

$$f(x)\,\phi(y) + \phi(x)\,f(y)$$

$$= \sin x \cos y + \cos x \sin y$$

$$= \sin(x+y)$$

$$= f(x+y)$$

Example 6

If $y = f(x) = \dfrac{ax+b}{cx-a}$ show that $x = f(y)$.

Solution

$$y = f(x) = \frac{ax+b}{cx-a}$$

$$y(cx-a) = ax + b$$

$$ycx - ay = ax + b$$

$$ycx - ax = ay + b$$

$$x(yc - a) = ay + b$$

$$x = \frac{ay+b}{cy-a} = f(y)$$

Exercise 1

1. If $f(x) = (x-1)(x-2)(x-3)$, find the values of $f(1)$, $f(4)$ and $f(-1)$.
2. If $f(x) = \tan x$ and $f(y) = \tan y$, prove that

$$f(x+y) = \frac{f(x) + f(y)}{1-f(x)\,f(y)}$$

3. If $f(x) = x^2 + x - 1$, simplify $f(x+1) - 3f(x) + 2f(x-1)$.
4. If $f(x) = x + \frac{1}{x}$, show that $f(x) = f\left(\frac{1}{x}\right)$

Linear Function

A function of the form $f(x) = ax + b$ where *a* and *b* are constants is called a linear function. The graph of the function $y = ax + b$ is a straight line in the xy plane. a is called the slope of the straight line and b is called the intercept, on the y-axis. If $b = 0$, the graph is a straight line passing through the origin. If $a = 0$, the graph is a straight line parallel to the x-axis and at a distance b units from the x-axis. A straight line can be expressed in the following forms :

(i) $y = mx + c$, called slope intercept form.

(ii) $\frac{x}{a} + \frac{y}{b} = 1$; here a and b are intercepts made on x and y axes.

(iii) $y - y_1 = m(x - x_1)$. This is the straight line passing through the point (x_1, y_1) and having a slope m.

(iv) $\frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2}$

This is the equation of the straight line passing through the points (x_1, y_1) and (x_2, y_2).

Let us now see a few problems on application of linear functions.

Example 1

The salary of an employee in 1985 was Rs 1200. In 1987, it will be Rs 1350. Express salary as a linear function of time and estimate his salary in 1988

(CA Nov 1986)

Solution

Let s represent the salary (in Rs) and t represent the year.

Year		Salary (Rs)	
1985	(t_1)	1200	(s_1)
1987	(t_2)	1350	(s_2)
1988	(t)	?	(s)

The equation of the straight line representing salary as a linear function of time is

$$s - s_1 = \frac{s_1 - s_2}{t_1 - t_2}(t - t_1)$$

$$\text{i.e.} \quad s - 1200 = \frac{1200 - 1350}{1985 - 1987}(t - 1985)$$

$$\text{i.e.} \quad s = 1200 + \frac{(-150)}{(-2)}(t - 1985)$$

$$= 1200 + 75\,(t - 1985) \qquad (1)$$

When t = 1988,

$$s = 1200 + 75\,(1988 - 1985)$$

$$= 1200 + 225$$

$$= 1425$$

∴ The estimated salary in 1988 is Rs 1425.

Example 2

The life expectancy of males in 1987 in a country is 70 years. In 1962, it was 60 years. Assuming the life expectancy to be a linear function of time, make a prediction of the life expectancy of males in that country in 1997 (CA May 1987)

Solution

Let E be the life expectancy and t the year.

Year		Life expectancy	
1962	(t_1)	60	(E_1)
1987	(t_2)	70	(E_2)
1997	(t)	?	(E)

The linear function representing the life expectancy in the year t is

$$E - E_1 = \frac{E_1 - E_2}{t_1 - t_2}(t - t_1)$$

i.e.

$$E - 60 = \frac{60 - 70}{1962 - 1987}(t - 1962)$$

$$E - 60 = \frac{2}{5}(t - 1962)$$

When $t = 1997$,

$$E - 60 = \frac{2}{5}(1997 - 1962)$$

$$E = 60 + \frac{2}{5} \cdot 35$$

$$= 74$$

∴ The predicted life expectancy in 1997 is 74 years.

Example 3

When the price is Rs 500, 50 cameras of a fixed type were available for sale. when the price was Rs 750., 100 of the cameras were available. What is the supply equation assuming that it is linear. If 150 cameras are made available, what is the expected price per camera?

Solution

Let y represent the price per camera and x the number of cameras available for supply.

Price Rs.	No of cameras available.
500 (y_1)	50 (x_1)
750 (y_2)	100 (x_2)
? (y)	150 (x)

The linear equation for price in terms of number of cameras is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

i.e. $$y - 500 = \frac{750-500}{100-50}(x-50)$$

$$= \frac{250}{50}(x-50)$$

$$y = 500 + 5x - 250$$

$$= 5x + 250$$

When $x = 150$,

$$y = 750 + 250 = 1000$$

∴ The price per camera is Rs 1000/–

Example 4

The demand curve for a commodity is $x = 10 - \frac{y}{4}$ where y is the price per unit and x is the number of units demanded.

(i) Find the quantity demanded if the price is Rs 4/-

(ii) Find the price if the quantity demanded is 7.

(iii) What quantity would be demanded if the commodity were free?

Solution

The linear relationship between price and quantity is

$$x = 10 - \frac{y}{4}$$

(i) When $y = 4$, $x = 10 - 1 = 9$

∴ When price per unit is Rs 4, the number of units demanded is 9.

(ii) When $x = 7$, $7 = 10 - \frac{y}{4}$ ∴ $y = 12$

i.e. When the number of units in demand is 7, the price per unit is Rs 12.

(iii) When $y = 0$, $x = 10$

If the commodity were free, number of units demanded will be 10 units.

Exercise 2

1. In 1950, 20 % of the population was educated; 50% was educated in 1980. If P denotes the percentage of the population educated and t the number of years since 1950,

(i) Express P as a linear function of time t.

(ii) Using this relationship make a prediction of the percentage of population educated in the year 2000.

2. A firm produces 200 units of a product for a total cost of Rs 730 and 500 units of the product for a total cost of Rs 970. Assuming the cost curve to be linear, derive the equation of this straight line and use it to estimate the cost of producing 400 units of the product.

3. A firm produces 50 units of a good for Rs 320 and 80 units for Rs 380. Assuming that the cost function is linear, estimate the cost of producing 10 units.

4. A manufacturer produces 80 T.V. sets at a cost of Rs 2, 20,000 and 125 sets at a cost of Rs 2,87,500. Assuming the cost to be linear, find the linear function and use it to estimate the cost of 95 sets.

5. The total cost Rs y of producing x units of a commodity is given by the linear relation $y = 2.5x + 300$. Find the fixed cost and then estimate the cost of producing an additional unit of the commodity.

6. The supply curve of a commodity is $x = 11y - 1$ where y represents price and x represents quantity supplied. Find the (i) the price when the quantity supplied is 54 and (ii) the quantity when the price per unit is Rs 2.

7. The total expenses (y) of a mess are partly constant and partly proportional to the number of inmates (x) of the mess. The total expenses are Rs 1040 when there are 12 members in the mess and Rs 1600 for 20 members. Find (i) the linear relationship between y and x and (ii) the constant expenses and the variable expenses per member.

Quadratic Function

A function of the form $f(x) = ax^2 + bx + c$ where a, b, c are constants is called a quadratic function or a polynomial of degree 2.
A quadratic polynomial can be expressed as the product of two linear factors. Let us now study some of the properties of the quadratic function.

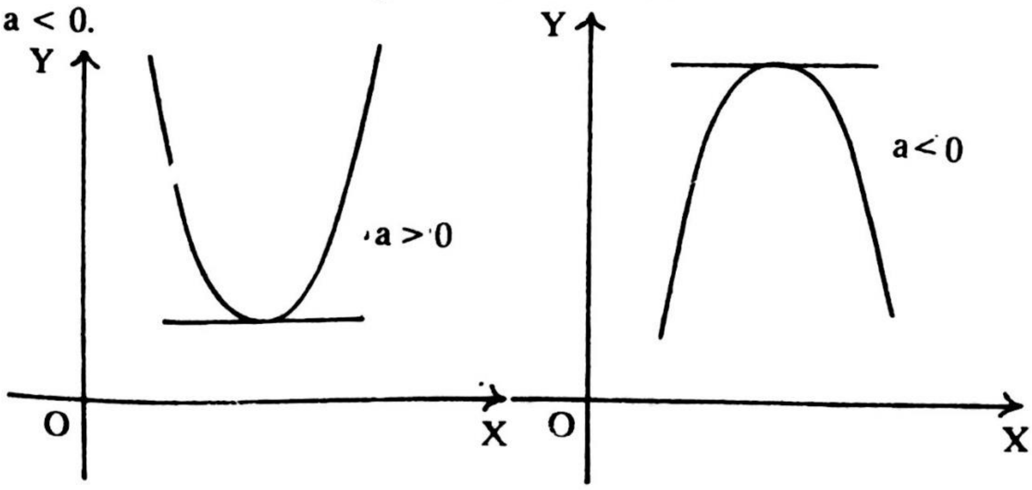
$$
\begin{aligned}
f(x) &= ax^2 + bx + c \\
&= a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] \\
&= a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2}\right] \\
&= a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2}\right] \\
&= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} \qquad (1)
\end{aligned}
$$

If a is *a* positive number, f(x) takes its minimum value at $x = \frac{-b}{2a}$

If *a* is negative f(x) takes its maximum value at $x = \frac{-b}{2a}$

The graph of the function $f(x) = ax^2 + bx + c$ is called a parabola. The point at which the minimum or maximum value occurs is called the vertex of the parabola. The vertex is $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

where $D = b^2 - 4ac$. We give the graph of f(x) when $a > 0$ and when $a < 0$.



Thus given a second degree polynomial $ax^2 + bx + c$, to find the maximum or minimum point on the graph of the function, we proceed as follows:

Step I

Find the values of a, b and c.

Step II

Determine whether $a > 0$ or $a < 0$

Step III

If $a > 0$, the minimum point occurs at $x = \frac{-b}{2a}$. Substituting this value of x in f(x) we obtain the minimum value of f(x).

If $a < 0$, the maximum point occurs at $x = \frac{-b}{2a}$. Substituting this value of x in f(x) we obtain the maximum value of f(x).

Roots of the quadratic equation

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$$

The roots of $ax^2 + bx + c = 0$ are given by

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus the roots are

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Note:

(1) If $b^2 - 4ac > 0$, the roots are real.

(2) If $b^2 - 4ac = 0$, the roots are real and equal.

(3) If $b^2 - 4ac < 0$, the roots are imaginary.

Example 1

The total profit in rupees of a drug company from the manufacture and sale of x drug bottles is given by

$$y = \frac{-x^2}{400} + 2x - 80$$

(i) How many drug bottles must the company sell to achieve the maximum profit?

(ii) What is the profit per drug bottle when this maximum is achieved?

(CA Nov 1985)

Solution

$$y = \frac{-x^2}{400} + 2x - 80 \qquad (1)$$

Comparing this with the quadratic function

$$y = ax^2 + bx + c \text{ we get,}$$

$$a = \frac{-1}{400},\ b = 2,\ c = -80$$

Here a is negative. Therefore y is maximum when

$$x = \frac{-b}{2a} = \frac{-2}{-2 \cdot \frac{1}{400}} = 400.$$

∴ To get maximum profit, the company should produce and sell 400 bottles.

The maximum profit is given by substituting $x = 400$ in (1)

$$\text{Maximum profit} = -\frac{400^2}{400} + 2 \cdot 400 - 80 = 320.$$

$\therefore$ Maximum profit per bottle $= \frac{320}{400} =$ Rs 0.80

Example 2

Let the cost function of a firm be given by the following equation

$$C(x) = 300x - 10x^2 + \frac{1}{3}x^3 \quad \text{where}$$

C (x) stands for cost function and x for output. Calculate :

(i) Output at which marginal cost is minimum.

(ii) Output at which average cost is minimum.

(iii) Output at which average cost is equal to marginal cost.

(CA Nov 1987)

Solution

$$C(x) = 300x - 10x^2 + \frac{1}{3}x^3$$

(i) The marginal cost function is given by

$$\frac{dC}{dx} = MC(x)$$

$$\therefore \quad MC(x) = x^2 - 20x + 300$$

This is a quadratic equation of the firm

$$ax^2 + bx + c \text{ where } a = 1, b = -20, c = 300$$

Since a > 0, the marginal cost is minimum at

$$x = \frac{-b}{2a} = \frac{20}{2} = 10$$

$\therefore$ The marginal cost is minimum when the output is 10 units.

(ii) Average cost function $= AC(x) = \frac{C(x)}{x}$

$$= \frac{x^2}{3} - 10x + 300$$

Here $a = \frac{1}{3}$, $b = -10$, $c = -300$

Since $a > 0$, AC is minimum at $x = \frac{-b}{2a} = \frac{10}{2/3} = 15$

$\therefore$ AC is minimum when the output is 15 units.

(iii) When AC = MC we have.

$$\frac{x^2}{3} - 10x - 300 = x^2 - 20x + 300$$

i.e. $$\frac{2x^2}{3} - 10x = 0$$

$\therefore$ $x = 0$ or $x = 15$

$\therefore$ The output at which MC = AC is 10 units.

Example 3

The price p per unit at which a company can sell all that it produces is given by the function $p(x) = 300-4x$. The cost function is $C(x) = 500 + 28x$, where x is the number of units produced. Find x so that the profit is maximum. (CA Nov 1987)

Solution

$$C(x) = 500 + 28x$$

$$p(x) = 300 - 4x$$

$$\text{Revenue} = x \cdot p(x) = 300x - 4x^2$$

The profit function is $P(x) = R(x) - C(x)$

$$= 300x - 4x^2 - 500 - 28x$$

$$= -4x^2 + 272x - 500$$

Comparing with the quadratic function $ax^2 + bx + c$,

$$a = -4, \quad b = 272, \quad c = -500$$

Since $a < 0$, profit is maximum when

$$x = \frac{-b}{2a} = \frac{-272}{-8} = 34$$

$\therefore$ For maximum profit the company should produce and sell 34 units.

Example 4

A manufacture finds that the cost per unit of manufacturing a certain commodity is $y = x^2 - 20x + 200$, when 5 to 200 units are produced per day. Find the number of units to be produced when the cost is least and also find the cost per unit.

Solution

The cost function is given by

$$y = x^2 - 20x + 200 \qquad (1)$$

This is a quadratic function of the form $ax^2 + bx + c$.

$$a = 1, \; b = -20, \; c = 200$$

Since $a > 0$, the cost per unit is minimum when

$$x = \frac{-b}{2a} = \frac{20}{2} = 10$$

For minimum cost the manufacturer should produce 10 units.

Cost per unit is got substituting $x = 10$ in (i)

$$y = 10^2 - 20 . 10 + 200 = 100.$$

$\therefore$ Minimum cost per unit = Rs 100.

Example 5

Find a quadratic function $y = ax^2 + bx + c$ that fits the data points (1. 4), (–1, –2) and (2, 13). Estimate the value of y when $x = 3$.

(CA May 1985)

Solution

$$y = ax^2 + bx + c \qquad \text{I}$$

This fits the data points (1, 4) , (–1, –2) and (2, 13). Since the points lie on the graph of I , we have,

$$4 = a + b + c \qquad (1)$$

$$-2 = a - b + c \qquad (2)$$

$$13 = 4a + 2b + c \qquad (3)$$

$(1)-(2): \quad 2b = 6 \qquad \therefore \; b = 3$

$(3)-(2): \quad 3a + 3b = 15$

$3a = 15-9 = 6$

$\therefore \quad a = 2$

From (1), $\quad 4 = 2 + 3 + c \qquad \therefore \; c = -1$

$\therefore$ The quadratic function is $\quad y = 2x^2 + 3x - 1$

When $x = 3$, $\quad y = 18 + 9 - 1 = 26$

$\therefore$ The estimated value of y when $x = 3$ is 26.

Exercise 3

1. Find a quadratic function $y = ax^2 + bx + c$ that fits the data points (1, 0), (2, 9) and (–2, –3). Estimate the value of y when $x = 4$.

2. Determine a quadratic polynomial $px^2 + qx + r$ given that it takes the values 2, 6, and 46 when $x = 1, 2$ and -3 respectively. What is the value of the function when $x = 4$?

3. A business man feels that the profits he can earn weekly is approximately a quadratic polynomial. During the first 3 weeks he earned Rs 550, Rs 960 , and Rs 1400. Fit a quadratic function for the data and estimate his profit in the fourth week.

4. Determine the profit of maximum or minimum value of the following functions :

(i) $-2x^2 + 8x + 3$

(ii) $3x^2 - 12x + 5$

5. A small manufacturing company produces and sells 10 to 50 units of its product per day. Its revenue function in rupees is given by $y = -x^2 + 100x$, where x is the number of units produced and sold. Obtain the number of units for which the revenue is maximum and obtain the maximum revenue

6. The electric power P (in watts) in a 240 volt line having a resistance of 20 ohms is given by the formula $P = 240a - 20a^2$ where a (in

Break Even Analysis

In business and economics to analyse the implications of various pricing and production decisions certain functions are designed. They are (i) Cost function (ii) Revenue function and (iii) Profit function. The costs have been divided into two categories – fixed cost and variable cost. Fixed cost remains constant at all levels of output and commonly includes such items as rent, depreciation, interest, plant and equipment, insurance and overhead expenses. Variable costs are those costs which vary with output and include such items as labour, materials and promotional expenses. Total cost at any level of output is the sum of the fixed cost and variable cost at the level of output.

i.e. Total cost function is $C(x) = F(x) + V(x)$

Here $C(x)$ is the total cost of producing x units of output.

$F(x)$ is the fixed cost of production of x units.

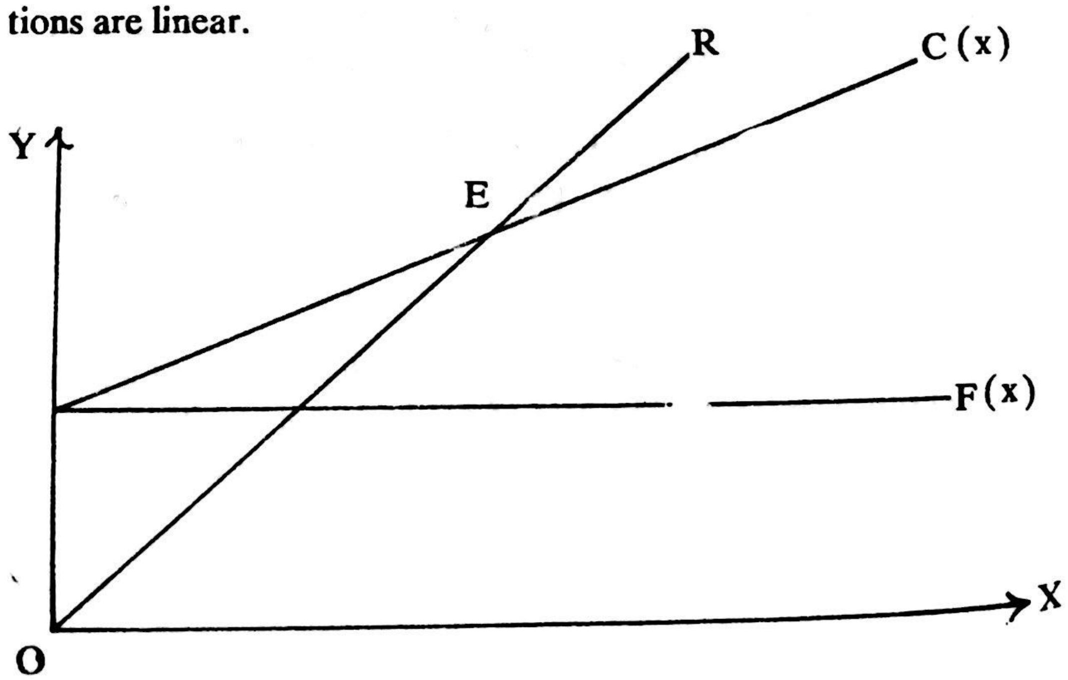
$V(x)$ is the variable cost of producing x units.

If p rupees is the selling price per unit, the revenue function for selling x units of output is given by $R(x) = x\,p$

The profit function $P(x)$ is given by $P(x) = R(x) - C(x)$

If $R(x) = C(x)$ then x is called the break-even quantity.
For break-even value x, $P(x) = 0$

Below we represent the graphs for fixed cost function, total cost function and total revenue function. We assume that all these functions are linear.



In the above figure, the straight line F(x) represents the fixed cost. For this straight line the y intercept is constant and this constant value is the fixed cost (namely the cost when the output is zero). The slope of this straight line is zero. The straight line C(x) represents the total cost, y – intercept is the fixed cost and its slope is the increase in variable cost per unit increase in output.

The straight line R(x) represents the graph of revenue function for different quantities sold. This straight line passes through the origin which means revenue is zero when output is zero. Its slope is the price per unit.

The point E at which the straight lines representing cost function C(x) and revenue function R(x) meet is called the Break even point – that is the revenue for this level of output is just sufficient to cover the production cost. In the above graph we have assumed that the functions C(x) and R(x) are linear. It may happen that these function may be quadratic or higher degree polynomials. In such situations the profit function P(x) may also be a quadratic or higher degree polynomial. In such cases, there can be more than one break–even value.

Example 1

For a manufacturer of dry cells, the daily cost of production C for cells is given by,

$$C(x) = \text{Rs } 2.05\,x + \text{Rs } 550.$$

If the price of a cell is Rs 3, determine the minimum number of cells that must be produced and sold to ensure no loss.

(CA May 1985)

Solution

The cost function is $C(x) = 2.05\,x + 550$

The revenue function for producing and selling x cells = Rs 3 x

$\therefore$ The profit function is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 3x - 2.05\,x - 550 \\ &= 0.95\,x - 550 \end{aligned}$$

For no loss, $P(x) \geq 0$; $\quad 0.95\,x - 550 \geq 0$

$\therefore \quad x \geq \frac{550}{0.95} = 578.9$

$\therefore$ For no loss the company should produce and sell at least 579 dry cells.

Example 2

A book publisher finds that the production cost of a book is Rs 30 and the fixed cost per year amounts to Rs 25,000. If each is sold at the rate of Rs 50 find

(i) Cost function (ii) the revenue function

(iii) the minimum number of books to be sold per year in order that there is no loss.

(CA Nov 1985)

Solution

Let x books be published and sold in a year.

Fixed cost $F(x) = \text{Rs } 25000$

Variable cost $V(x) = 30x$

The total cost of publishing x books is

$C(x) = F(x) + V(x)$

$C(x) = 25000 + 30x$

The revenue function is given by,

$R(x) = 50x$

The profit function is given by

$P(x) = R(x) - C(x)$

$R(x) = 50x - 25000 - 30x$

$= 20x - 25\,0000$

For no loss, $P(x) \geq 0$ i.e. $20x - 25000 > 0$

$\therefore \quad x \geq 1250.$

$\therefore$ To ensure no loss, at least 1250 books should be sold in a year.

Example 3

The daily cost of production c for x units of an assembly is given by

$$C(x) = \text{Rs } 12.5x + 6,400$$

(i) If each unit is sold for Rs 25, determine the minimum number of units that should be produced and sold to ensure no loss.

(ii) If the selling price is reduced by Rs 2.5 per unit, what would be the break-even point?

(iii) If it is known that 500 units can be sold daily, what price per unit should be charged to guarantee no loss?

(CA Nov 1986)

Solution

The cost function is given by

$$C(x) = \text{Rs } (12.5x + 6400)$$

The revenue function is given by

$$R(x) = \text{Rs } 25x$$

The profit function is $P(x) = R(x) - C(x)$

$$P(x) = 25x - 12.5x - 6400$$
$$= 12.5x - 6400$$

(i) To ensure no loss, $P(x) \geq 0$

$\therefore \quad 12.5x - 6400 \geq 0$

i.e. $\quad x \geq \frac{6400}{12.5} = 512$

$\therefore$ To ensure no loss at least 512 units should be produced and sold.

(ii) When the selling price is reduced by Rs 2.50 per unit
$R(x) = 22.5x$

For break-even point, $R(x) = C(x)$

$$22.5x = 12.5x + 6400$$
$$10x = 6400$$
$$x = 640$$

$\therefore$ For break – even, 640 units should be produced and sold.

(iii) Let p be the selling price per unit.

The revenue by sale of 500 units = 500 p

The cost of producing 500 units is

$$C(500) = 12.5 \times 500 + 6400$$
$$= \text{Rs } 12,650$$

To guarantee no loss, $500\,p \geq 12650$

$$p \geq \frac{12650}{500} = 25.30$$

If 500 units can be sold, unit price should be fixed as Rs 25. 30 to guarantee no loss.

Example 5

A company sells x tins of talcum powder each day at Rs 10 per tin. The cost of manufacturing is Rs 6 per tin and the distributor charges re 1 per tin. Besides, these the daily overhead cost comes to Rs 600. Determine the profit function. What is the profit if 500 tins are manufactured and sold a day ? How do you interpret the situation if the company manufactures and sells 100 tins in a day ? What is the break-even point?

(CA May 1987)

Solution

Variable cost per unit = Rs (6 + 1) = Rs 7

Fixed cost = Rs 600

$\therefore$ The cost of producing x tins is given by

$$C(x) = 7x + 600$$

The revenue function is $R(x) = 10x$

The profit function

$$P(x) = R(x) - C(x)$$
$$= 10x - 7x - 600$$
$$= 3x - 600$$

When x = 500, $P(x) = 3 \,.\, 500 - 600$ = Rs 900.

The profit in producing and selling 500 units = Rs 900.

(ii) When 100 tins are produced and sold

$$P(100) = 3 \,.\, 100 - 600 = -300$$

$\therefore$ The company will incur a loss of Rs 300.

(ii) For Break – even point, P(x) = 0

$$3x - 600 = 0 \qquad \therefore x = 200$$

$\therefore$ For break-even the company should produce and sell 200 units

Example 6

A T.V. manufacturer determines that his total cost for producing x TV sets is given by $C(x) = 500x^2 + 2500x + 5000$

Each set sells for Rs 6000. Determine(i) the break-even points, (ii) the number of sets to be produced for no loss and (iii) the number of sets to be produced for no profit.

Solution

The cost function is

$$C(x) = 500x^2 + 2500x + 5000$$

The revenue function $R(x) = 6000x$

(i) For break –even, C(x) = R(x)

$$500x^2 + 2500x + 5000 = 6000x$$

$$500x^2 - 3500x + 5000 = 0$$

$$x^2 - 7x + 10 = 0$$

$$(x-2)(x-5) = 0$$

$$\therefore \quad x = 2 \text{ or } 5$$

$\therefore$ For break–even, the manufacturer should produce 2 sets or 5 sets.

(ii) The profit function is P(x) = R(x) – C(x)

$$P(x) = 6000x - 500x^2 - 2500x - 5000$$

$$= -500x^2 + 3500x - 5000$$

For no loss, $P(x) \geq 0$

i.e. $-500x^2 + 3500x - 5000 \geq 0$

$x^2 - 7x + 10 \leq 0$

$(x-2)(x-5) \leq 0$

$\therefore \quad 2 \leq x \leq 5$

To incur no loss, the manufacturer should produce 2 to 5 sets.

(ii) For no profit, $P(x) \leq 0$

$\therefore \quad -500x^2 + 3500x + 5000 \leq 0$

$(x-2)(x-5) \geq 0$

$5 \leqslant x \leqslant 2$

The manufacturer will incur no profit if he nroduces 2 sets or less or 5 sets or more.

Exercise 4

1. The fixed cost of production for a commodity is Rs 5000.The variable cost is Rs 750 per unit.The company can sell Rs 10 per unit. What is the break-even quantity ?

2. A manufacturer sells his product at Rs 5 per unit. Fixed cost of production is Rs 3000. The variable costs are estimated at Rs 40 % of total revenue (i) Find the total cost function (ii) the break-even point.

3. The fixed cost of production for a commodity is Rs 90,000 The variable cost 60% of the selling price of Rs 30 per unit. What is the break-even quantity ?

4. The cost function for producing x units of a commodity is given by $C(x) = 30x + 300$. The revenue function is given by $R(x) = 100x - 2x^2$. Find (i) the break-even point (ii) the value of x that results in a loss (iii) the value of x that results in a profit.

5. A text-book publisher finds that the production cost directly related to each book is Rs 40 and that the fixed cost is Rs 15,000. If each book can be sold for Rs 60 find (i) the cost function (ii) the revenue function and (iii) the break–even point.